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Chilean Coins and Banknotes

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FORECASTING DEMAND FOR DENOMINATIONS OF CHILEAN COINS AND BANKNOTES[*]

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Abstract

Monetary authorities have to plan how many units of coins and bank notes they need to buy / produce to meet the needs of the economy. With data from the Central Bank of Chile, the process of supplying coins and banknotes to the banking sector is described. This is followed by exercises aiming at finding useful models for forecasting cash demand for the horizons up to three years, which are the relevant ones for planning purchases and delivery of individual banknotes in Chile. With respect to the aggregate circulating stock of cash, time series models seem to perform better for the one–year–ahead horizon, while models based on fundamental variables are better for two and three years. Time series models for each denomination seem to be the best forecasting option to guide the purchases of notes and coins.

Resumen

Las autoridades monetarias tienen que planificar la cantidad de monedas y billetes que deben comprar/producir para satisfacer las necesidades de la economía. Usando datos del Banco Central de Chile, se describe el proceso de suministro de monedas y billetes al sector bancario. Luego se realizan ejercicios que buscan encontrar modelos útiles para predecir la demanda de efectivo para los horizontes de hasta tres años, período relevante para la planificación de compras y entregas de billetes en Chile. Para el agregado de efectivo circulante, los modelos de series de tiempo muestran en general un mejor desempeño para el horizonte de un año, mientras que los modelos basados en variables fundamentales muestran mejores resultados para los horizontes de dos y tres años. Con respecto a las proyecciones por denominación, los modelos de series de tiempo parecen ser la mejor opción para guiar las compras de billetes y monedas.

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1. Introduction

Several studies are concerned with the analysis of the demand for money and the focus is usually on broad measures such as M1 and M3.[1] There are fewer studies focused on the demand for cash even though plausible forecasts of future demand for banknotes and coins are important inputs for efficient stock management, a concern of the monetary authorities in several countries. Cash stock management is no trivial issue, as the monetary authorities have to balance the costs of having too large a stock of cash against the risk of not being able to supply what the economy demands. The cost of the latter is, however, not directly measurable. The investigation in the present paper pursues the quest of finding suitable models for predicting the change in the stock of outstanding notes and coins in Chile, which in turn may help to plan future orders and deliveries and, hence, to optimize the stock of cash the Central Bank needs to maintain.[2]

More concretely, we extend the work of Chumacero et al. (2008) by presenting some new time series models as well as some that include fundamental variables,[3] and evaluate the forecast performance with respect to total stock of cash and for each denomination. As there are several models, we also consider the average of the forecasts. In this context, a question of interest is whether models based on fundamental variables[4] perform better than time series models when out-of-sample forecasts of cash demand are compared. This is particularly

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[1] Surveys of the money demand literature are offered by e.g. Barnett et al. (1992) and Duca and VanHoose (2004).

[2] The issue of minimizing inventory costs is a complex one, and the present research paper merely presents a number of econometric models, which can be utilized in the forecasting process to improve the estimations of future demand.

[3] The models based on fundamentals are inspired by Baumol (1952) and Tobin (1956) who find a relation between money demand, activity and the opportunity cost of holding money, i.e., the interest rate. Even though the elasticities in the Baumol-Tobin model may be better captured by broader measures of money, in the present paper we estimate elasticities with respect to the demand for cash. Briglevics and Schuh (2014) employ an extended version of the Baumol-Tobin model (Sastry, 1970) that allows for the use of credit cards to study U.S. consumer demand for cash.

[4] We limit ourselves to considering national accounts variables, i.e. the gross domestic product and private consumption, as indicators of Chilean activity.

interesting for aggregate cash demand and, even though the denomination forecasts are the relevant ones for the planning of future purchases, aggregate forecasts provide relevant information for applying judgement of the predictions in order to finally decide how many pieces should be ordered.

A priori one might expect that models based on fundamental variables would perform better, when the money supply is endogenous, whereas time series models might be more appropriate for economies where it is exogenous. In the case of Chile, and this is probably the case for other economies as well, the supply of banknotes is best described as being exogenous during the period investigated in the sense that not all of the banks' demand for bills has been supplied. On the other hand, given this fact, the commercial banks may inflate their demand in order to obtain the quantity of bills they desire given the restriction they expect the central bank to impose. Hence, in practice the Chilean supply may be characterized as being exogenous during the period analyzed, but with an important endogenous component.

The evidence from the forecasting exercises suggests that fundamental-based models forecast aggregate cash demand better than time series models in the long run (two and three years ahead), which implies that forecasts of fundamental variables should be taken into account when placing orders for the pieces of denominations. Forecasts by denomination are better made with time series models, but the type of model varies across the different coins and banknotes.

Amongst the existing studies of currency demand,[5] only a handful is concerned with forecasting. Gerst and Wilson (2011) estimate models for each dollar bill denomination up

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[5] Recent literature on currency demand includes the studies of Doyle (2000): Foreign demand for currencies of the US, Germany, and Switzerland; Khamis and Leone (2001) on real currency demand in Mexico after its financial crisis in 1994; Akinci (2003) on currency circulation in Turkey; Fischer et al. (2004) on circulation of euro; Amromin and Chakravorti (2009) on demand for small currency denominations in 13 advanced economies; Columba (2009) on transaction technology and demand for currency and M1 in Italy; Cusbert and Rohling (2013) on currency demand in Australia during the recent international financial crisis; Bagnall et al. (2014) conduct a comparison of consumer cash usage in seven industrialized countries; Briglevics and Schuh

to US$100 as well as aggregate demand with models that include information of regulation / policy, domestic and international macroeconomic variables, demographic trends, and technology and consumer taste. They also include seasonal dummies in the model and with a pseudo out-of-sample forecast exercise, using actual values of the exogenous variables, they argue that the model does a "fairly good job of predicting." To predict the demand for several denominations of Norwegian coins and notes, Vale (2015) estimates vector error correction models including as exogenous variables point-of-sale consumption by the household sector (goods and services for which consumers are likely to consider using notes and coins), number of terminals for electronic funds transfer at points of sales, the central bank's interest rate and some specific dummies including seasonal dummies. He argues that these model out-of-sample forecast "reasonably well" over the horizon of eight quarters. For Chile, Chumacero et al. (2008) employ autoregressive models and autoregressive distributed lag models to estimate demands for real stock (deflated by the consumer price index) of coins and notes. The explanatory variables are monthly data of GDP and an interest rate. They find that the univariate time series models make the better predictions, but do not supply the specifications of the best models.

The approach employed in the present study is different from those mentioned above in a couple of ways. Firstly, we forecast aggregate demand for Chilean coins and notes with time series models as well as models with fundamental macroeconomic variables. As an additional exercise, we obtain forecasts by denomination, where the weights are estimated with autoregressive models. Secondly, several time series models are estimated and forecast performance is also evaluated for the average of these models. In this sense, the present analysis supplies a number of new models, which can be employed in the forecasting process taking place before planning the orders of coins and notes.

The next section offers some stylized facts of the aggregate stock of banknotes in Chile and discusses the relation between the commercial banks' demand for bills and the supply of the

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(2014) on US consumer demand for cash; Dunbar (2014) on currency demand and demographics in Canada; and Bartzch et al. (2015) investigate demand for euro banknotes in Germany.

central bank. Section 3 presents the models explored, while section 4 presents the results of the forecasting exercise. Finally, section 5 offers some concluding remarks.

2. Banknotes in Chile: Stylized facts, demand and supply

The Central Bank of Chile (CBCh) has the exclusive faculty to issue banknotes and coins, regulating the amount of currency in circulation. The current family of banknotes was put into circulation between 2009 and 2011, and it is printed on two types of material. The $1,000, $2,000 and $5,000 peso bills are printed on polymer substrate, while the $10,000 and $20,000 bills are printed on cotton substrate. Since 2006 the printing programs of banknotes has been the result of public tenders.

To estimate the adequate amount of issued currency in the economy, the CBCh utilizes as guideline time series models[6] that project by nomination the stock in circulation one, two and three years ahead.[7] An accurate estimation of the future demand for cash is important for several reasons. First, the printing program has a significant cost for the Central Bank that annually averages about 80% of the operational costs. Second, due to logistical concerns and technological constraints, the planning and bidding process for printing notes begins around three years in advance, so the cost of adapting the program to a shorter term can be quite expensive. Third, the inventory costs associated with physical requirement and management should be minimized.

The purchases are made three years early for two main reasons. Firstly, given the relatively small size of the Chilean economy, for suppliers it may not be profitable to manufacture the notes to be utilized during a shorter period of time. Secondly, the bidding process can be very

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[6] See Chumacero et al. (2008).

[7] The bidding process for the manufacturing of banknotes starts three years in advance, the last contract was tendered in 2013 and ends in 2016. The first shipment of banknotes arrives nine months after the contract is signed and the CBCh implements final quality controls of this shipment. During the following months, new bills arrive according to the stipulations in the contracts.

extensive and bureaucratic, lasting sometimes up to five months; thus, it would be quite costly for the monetary authority to conduct the process too often.

2.1. Some stylized facts

Banknotes and coins in circulation, including the cash held by financial institutions, represent about 90% of the monetary base. As shown in Figure 1, this ratio was relatively stable during the late 1990s, but it started to fall in 2004 to reach around 76% in 2011. In 2014, it increased back to the 90% level. With respect to banknotes and coins, about 70% is circulating in the economy and the rest is held by commercial banks and financial institutions.

Figure 1: Banknotes and coins as percentage of monetary base

Source: Authors' elaboration with data from the Central Bank of Chile.

As for M1, over the same period of time, the percentage of banknotes and coins has fallen from an average of 38% in 1994 to 29% in 2015 (see Figure 2). Considering the broadest definition of money (M3), the percentage of banknotes and coins has been quite stable around 4.5%, but it increased steadily since 2000, reaching levels close to 5% since 2012 (see Figure 2).
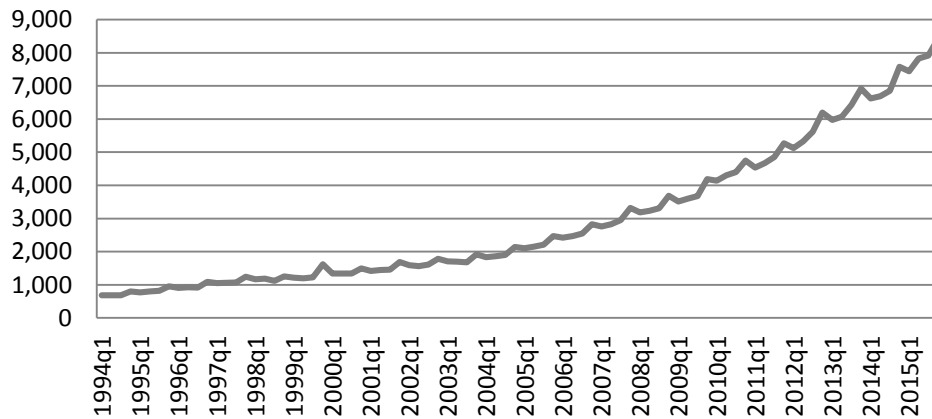
Figure 2: Banknotes and coins as percentage of money supply (M1 and M3)

Banknotes and coins as % of M1

Banknotes and coins as % of M3
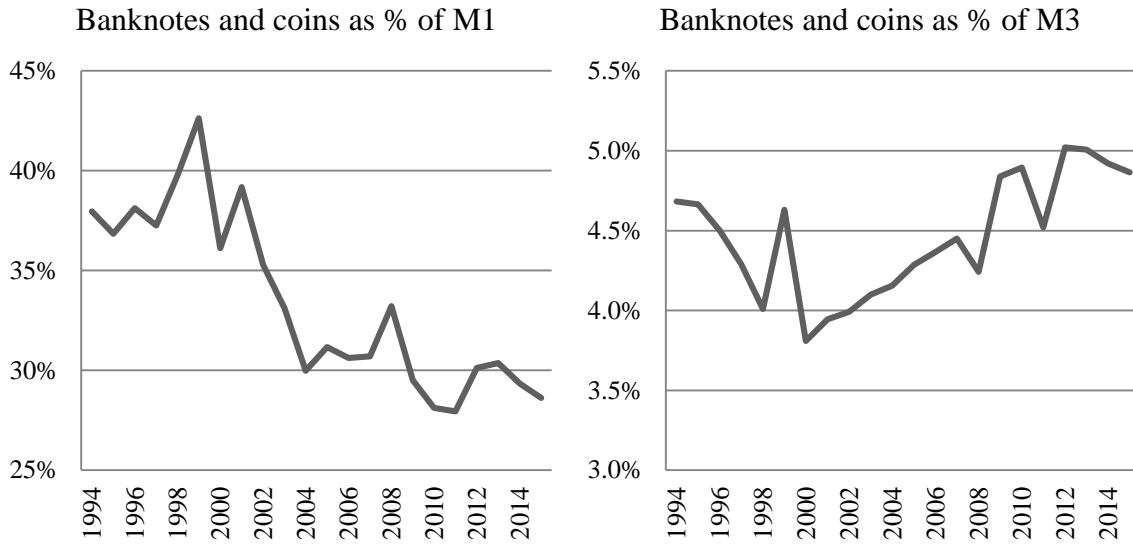
Source: Authors' elaboration with data from the Central Bank of Chile.

Figure 3. Total stock of banknotes and coins

(billions of Chilean pesos)

Source: Central Bank of Chile.

The total stock of banknotes and coins has more than tripled its nominal value over the last ten years (see Figure 3). With respect to the GDP growth rate –and with respect to private consumption– the stock of banknotes and coins has also increased, implying that in Chile

banknotes and coins are still important in the payment process despite the development of alternative ways of paying such as bank cards and electronic transfers[8] (see Figure 4).

Figure 4: Relative growth of outstanding stock of cash
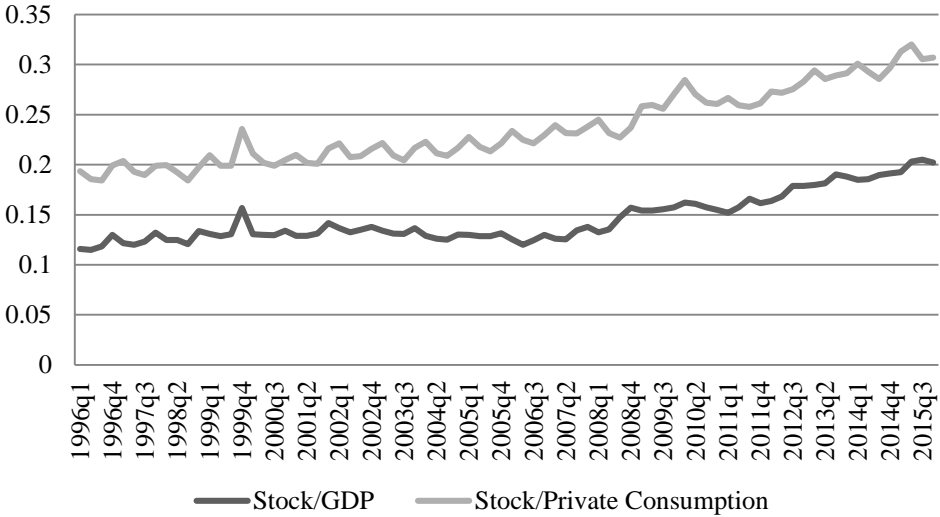
Source: Authors' elaboration using public data from the Central Bank of Chile.

The cycle of banknotes is summarized in Figure 5.[9] Once a month, commercial banks demand banknotes by denomination, which includes the demand for new pieces and the replacement of notes that are no longer suitable for circulation. At any point in time, the commercial banks may make deposits in the custody vaults of the CBCh located throughout the country. Deposits consist of suitable and non-suitable pieces, but only the suitable ones can be withdrawn, while non-suitable pieces will be kept by the CBCh until the entity decides to replace them through the monthly supply.[10]

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[8] From 2009 to 2015, the total volume of transactions using cards (including debit cards and bank and nonbank credit cards) increased from 17.4 to 27.3% of consumption according to Central Bank of Chile (2016).

[9] In the case of coins, there is no withdrawal or replacement of unfit pieces and the demand by commercial banks is always met. A more detailed illustration of the banknotes cycle is presented in figure 4 in Chumacero et al. (2008), while figure 1 in the same article shows the coin cycle.

[10] The CBCh analyzes through technical procedures the actual state of the pieces and replaces them if found unsuitable for circulation. The rate of replacement of each denomination depends on the inventory and projections of future demand. The rigidity of having to make long-termed contracts limits the possibility of

Figure 5: Cycle of banknotes
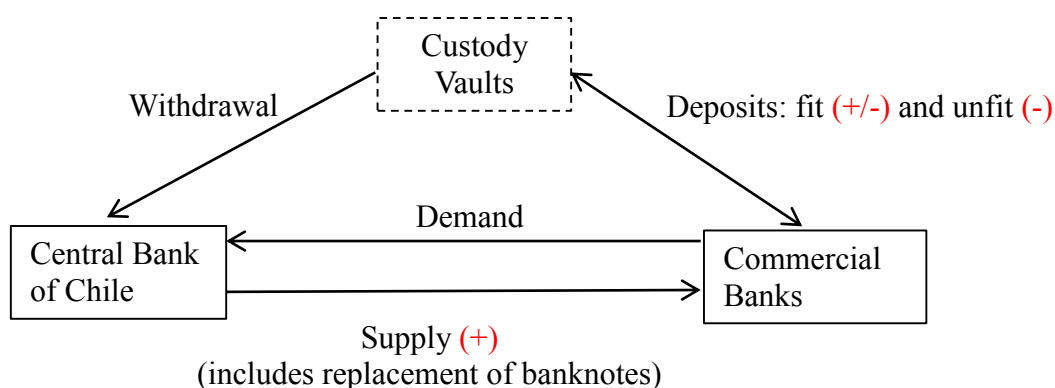
Source: Authors' elaboration.

This way the **stock** of banknotes —defined as the total banknotes and coins circulating and held by commercial banks and financial institutions— decreases (-) with deposits of suitable and non-suitable banknotes, and increases (+) with the monthly supply by the CBCh and withdrawals of banknotes from the custody vaults. It should be emphasized that about three quarters of the supply of banknotes is made to replace subtractions of notes that are no longer suitable for circulation. Hence, to obtain accurate predictions of future demand for banknotes, a study of e.g. the expected lifetime of the notes is needed, a subject which is beyond the scope of the present paper.

2.2. Demand and supply

As previously described, once a month the commercial banks request from the CBCh banknotes and coins (new issues as well as replacement), the amounts provided to the banks are deducted from their current account with the public entity. The CBCh usually supplies the demanded quantity of every denomination, or a lower amount, taking into account the inventory and projections of future demand.[11] While coins are always supplied as requested, the supply of banknotes has often been lower than the demand (see Figure 6). This fact may

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obtaining additional pieces in the short run. Hence, the most demanded denominations are usually longer in circulation, because of an effort to extend their useful lifetime as much as possible.

[11] When the demand is not met by the CBCh, the commercial banks may try to exchange denominations with other banks.

have been internalized by commercial banks, such that it cannot be ruled out that effective orders may sometimes exceed actual needs.

Figure 6: Demand for and supply of banknotes[12]

(billions of Chilean pesos)
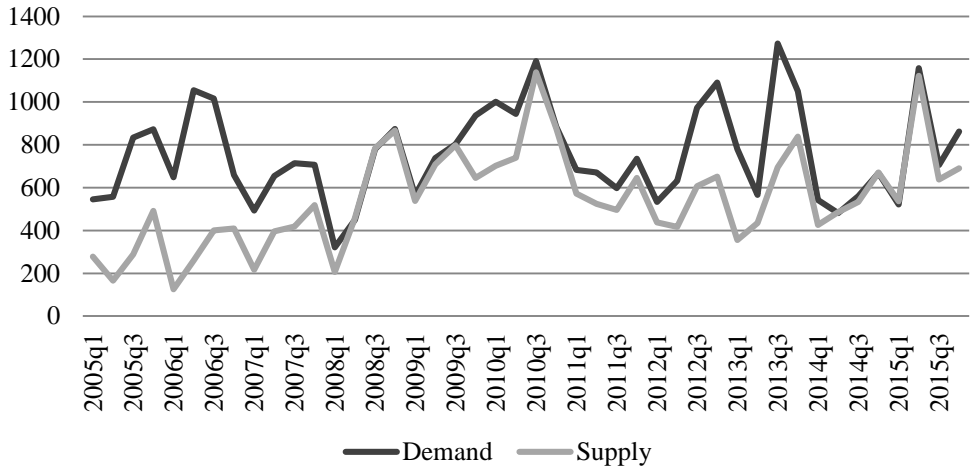
For some denominations the difference between demand and supply has been more persistent over time. In particular, for the $2,000 and $5,000 banknotes there were significant gaps during the 2005q1-2007q1 and 2011q1-2013q3 periods (see Figure 7). During the second period, the demand for both denominations reached around three times the effective supply. The persistence of these gaps could be present in order to compensate for previous gaps. The demand for $2,000 and $5,000 banknotes declined in 2013, which may be partly explained by the decision to remove the lower denominations ($1,000, $2,000 and $5,000) from the ATMs.

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[12] Monthly data are provided by the Department of Analysis and Circulating Supply of the CBCh.

Figure 7: Demand and supply by denomination

(millions of pieces)

$2,000

$5,000

$10,000
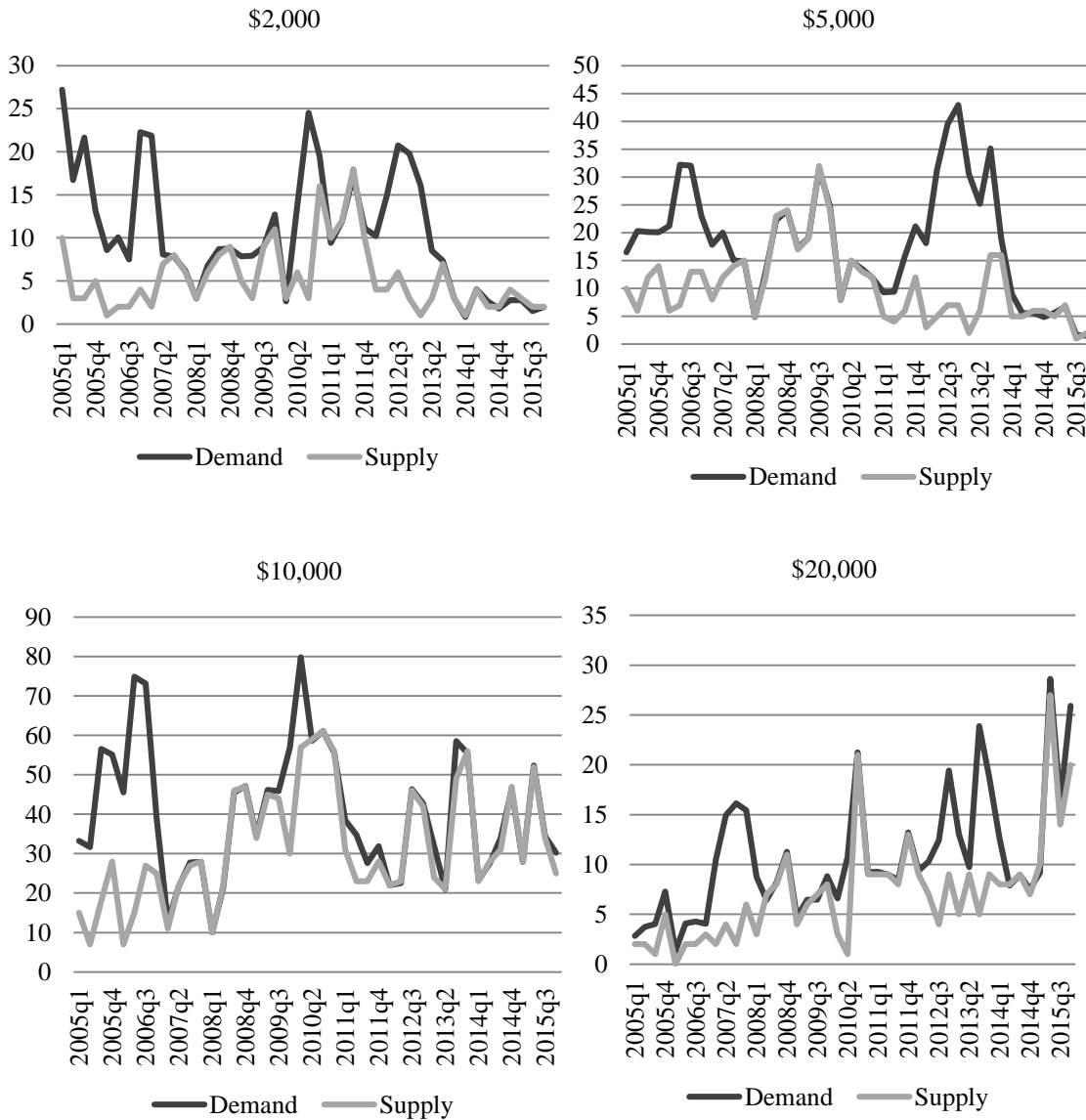
$20,000

Source: Central Bank of Chile.

ATMs with only high denomination banknotes imply a lower cost for banks, as they have to reload them with less frequency. This change has also implied a higher demand for $20,000 banknotes, a demand that could not always be satisfied (see Figure 7). The demand of $10,000 banknotes, however, remained relatively stable and was usually met.

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3. Data description and econometric models

This section discusses the data utilized and outlines the econometric models that are used in the forecasting exercise in section 4. The next subsection presents the data and discusses seasonality and stationarity. Subsections 3.2 and 3.3 present the time series models and those estimated with fundamental variables.

3.1. Data

The stock series (by denomination) for banknotes and coins are monthly, measured in millions of pieces, and are available from January 1985. The data were provided by the Department of Analysis and Circulating Supply of the CBCh. The series are published in the Statistical Database of the CBCh from 1994 onwards.

As mentioned earlier, a topic investigated in the present paper is whether fundamental variables contain useful information for planning the purchases of coins and banknotes in Chile. Following the Baumol (1952) and Tobin (1956) line of thinking, the fundamental variables considered in the present context are economic activity and the interest rates. With respect to activity, we apply data from the national accounts, more precisely the gross domestic product (GDP), and variables of private consumption, which may be more related to the demand for coins and banknotes. Beside the aggregate private consumption, it is also disaggregated into durables, nondurables, and services. These series are quarterly at current prices (millions of CLP) and are available from the first quarter of 1996. The data are published in the Statistic Database of the CBCh.[13]

The interest rate used in the analysis, to approximate the cost of money, is the monthly average nominal deposit rate of the financial system (30- to 89-day term, annual percentage). Data for the nominal deposit rate is available from January 1990, published in the Statistical Database of the CBCh. Estimations and projections are made with quarterly observations from 1996.

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[13] http://si3.bcentral.cl/Siete/secure/cuadros/home.aspx?Idioma=en-US.

3.1.1. Seasonality

The change in the stock of banknotes and coins presents a high degree of seasonality, aggregated and by denomination (see Figure 8 for total stock and Figures A.1 and A.2 in the Appendix for the stock by denomination). During the fourth quarter the circulating stock of cash has its biggest increase (Christmas and New Year effects), followed by a drop in the first quarter and smaller changes during the second and third quarters. In a simple exercise where the circulating stock is regressed on seasonal dummies, the results indicate than almost 76% of the total variation can be explained by seasonality.

Figure 8: Change in total stock of banknotes and coins

(billions of Chilean pesos)
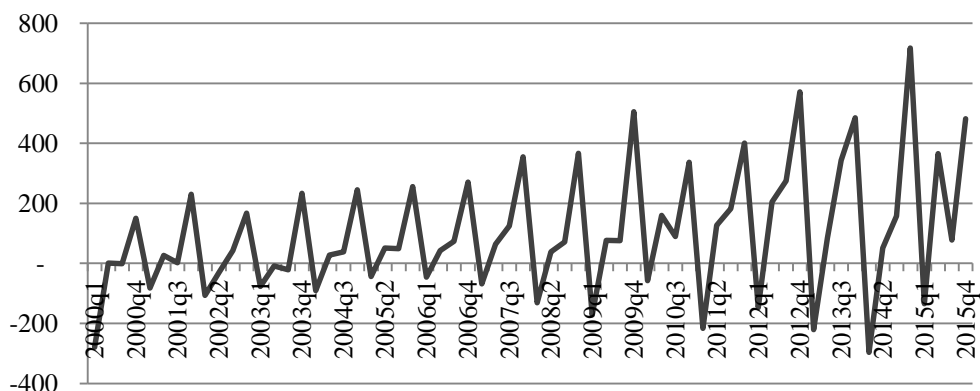
Source: Central Bank of Chile.

The same seasonality is naturally observed in total cash demand and supply (see Figure 6 for banknotes). Repeating the previous exercise, it is obtained that around 87% of the variation of total demand and total supply of banknotes and coins is explained by seasonality. By denomination, demand and supply present similar results but less pronounced than for the total (see Figure A.1. and A.2. in the Appendix for demand for and supply of banknotes and coins, respectively).

3.1.2. Unit roots

To tests for unit roots in the series applied, the Phillips-Perron test and the Augmented Dickey Fuller unit root tests are applied. The results (Table 1) suggest that the total circulating stock

of banknotes and coins and the activity proxies are integrated of order 1 (I(1)), though the results are mixed with respect to the deposit rate.

Table 1: Unit Root Tests (p-values)[14]

Variable	Augmented Dickey-Fuller		Phillips-Perron	
	Level	First difference	Level	First difference
Total stock	1.0000	0.0315	1.0000	0.0000
Deposit rate	0.4414	0.0002	0.0000	0.0000
GDP	0.3932	0.0113	0.6952	0.0000
Private consumption	0.8618	0.0000	0.9605	0.0000
Durable consumption	0.5935	0.0005	0.2716	0.0000
Nondurable consumption	0.9052	0.0000	0.8046	0.0000
Services consumption	0.8327	0.0088	0.9472	0.0000

Source: Authors' elaboration with data from the CBCh.

Applying the Johansen (1991) trace test, there is no indication that cointegration exists between the circulating stock and any of the fundamental variables, i.e. there does not appear to be any long-term relations present in the data.[15]

3.2. Time series models

We consider six different time series models, named Mod.1 to Mod.6. The first two are random walk types of model, where the seasonal pattern of the series is taken into account. Mod.1 simply postulates that the quarterly growth rate is the same as the one four quarters before, whereas the second estimates the growth rate in quarter t as the average of the rates of the same quarters the five previous years:

$$\Delta x_t = \Delta x_{t-4} + \varepsilon_t, \tag{Mod.1}$$

$$\Delta x_t = \frac{1}{5}\sum_{i=1}^{5} \Delta x_{t-4i} + \varepsilon_t, \tag{Mod.2}$$

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[14] The tests include a constant term, a trend and four lags of the difference of the variable. The null hypothesis states that the variable contains a unit root, while the alternative is that it is generated by a stationary process.

[15] The results are available upon request.

where ε_t are the error terms.

The models Mod.3 and Mod.4 are autoregressive, where the number of lags is determined by the Schwarz information criterion. The first includes a constant term, whereas the second also includes seasonal dummies.

$$\Delta x_t = c + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + \varepsilon_t, \qquad \text{(Mod.3)}$$

$$\Delta x_t = c + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + \sum_{j=1}^{3} \gamma_j s_j + \varepsilon_t, \qquad \text{(Mod.4)}$$

where c is a constant term, p is the lag length, β_i (i=1,2,...,p) and γ_j (j=1,2,3) are coefficients to be estimated, and s_j (j=1,2,3) are seasonal dummies.

Finally, two seasonal ARIMA models are included in the analysis. The first is the standard airline model of Box and Jenkins (1976), ARIMA(0,1,1)×(0,1,1)$_4$, while the second also includes seasonal AR components, ARIMA(1,1,1)×(1,1,1)$_4$.

$$(1-L)(1-L^4)x_t = (1-\theta L)(1-\Theta L^4)\varepsilon_t, \qquad \text{(Mod.5)}$$

$$(1-\phi L)(1-\Phi L^4)x_t = (1-\theta L)(1-\Theta L^4)\varepsilon_t, \qquad \text{(Mod.6)}$$

where L is the lag operator and θ, Θ, Φ, and ϕ are coefficients to be estimated.

3.3. Empirical models based on fundamentals

Theoretically, one expects a positive elasticity of money demand with respect to activity and negative with respect to the interest rate. As mentioned earlier, in the following empirical applications the activity is approached with the GDP and measures of private consumption, while the interest rate used is the short-term deposit rate. The dependent variable is the total circulating stock of banknotes and coins (measured in millions of Chilean pesos). Estimating the total value of circulating allows us to incorporate in the analysis the possible substitution between denominations.

3.3.1. Is the history useful for explaining variations in circulating stock?

The first approach is to measure the effect of past values of fundamentals on the change in total stock. We estimate the following simple bivariate VAR model:

$$X_t = \beta_0 + \sum_{i=1}^{4} \beta_i X_{t-i} + \sum_{i=1}^{3} \delta_i s_{it} + \varepsilon_t,$$

where β_0 is a constant term, $X_t = \begin{bmatrix} \Delta \ln(M_t) \\ \Delta \ln(Y_t) \end{bmatrix}$, M_t is the total stock of banknotes and coins, Y_t is the activity variable, and s_{it} are seasonal dummies.

The exercise suggests that it is not obvious that lags of the fundamental variables explain some of the variation in the contemporaneous circulating stock. Even though some lags do turn out to be statistically significant, it is never one of the first two lags, and often the sign of the coefficient is the opposite of the expected one (see Table A.1 in the Appendix). Hence, from this exercise we conclude that the history of the fundamental variables do not help to explain variations in the circulating stock.[16]

3.3.2. Contemporary models

To explore the contemporary relation between the total circulating stock and fundamental variable, we apply models of the following form:

$$\Delta\ln(M_t) = \beta_0 + \beta_1 \ln(X_t) + \sum_{i=1}^{3} \delta_i s_{it} + \varepsilon_t,$$

where $X_t = \begin{bmatrix} \Delta \ln(Y_t) \\ \Delta \ln(1 + i_t) \end{bmatrix}$, Y_t is one of the activity variables and i_t is the short term deposit rate. The results presented in Table 2 show that private consumption, non-durable consumption and consumption of services are statistically significant. The deposit rate by itself is also statistically significant and has the expected sign; however, when interacting with the activity variable it becomes statistically insignificant.

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[16] The VAR models were also employed to forecast, but the results were inferior to the ones reported in the present study.

Table 2: Regressions of the Quarterly Variation of Total Stock

Variable	Δln(Stock)	Δln(Stock)	Δln(Stock)	Δln(Stock)	Δln(Stock)
Δln(GDP)	0.092				
	(0.616)				
Δln(Private Cons.)		1.004***			
		(0.166)			
Δln(Non-durable Cons.)			0.731***		
			(0.142)		
Δln(Services Cons.)				0.663**	
				(0.265)	
Δln(1+Deposit rate)	-0.0015	-0.351	-0.322	-0.161	-0.339***
	(0.248)	(0.232)	(0.221)	(0.247)	(0.102)
Constant	0.112***	0.046**	0.036***	0.106***	0.134***
	(0.017)	(0.174)	(0.012)	(0.009)	(0.007)
Observations	79	79	79	79	103
R-squared	0.748	0.803	0.815	0.767	0.766

Source: Authors' elaboration with data from the CBCh.
Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

4. Forecasting

To evaluate the time series models and fundamental based ones, we estimate them with data from 1996q1 to 2008q4 and then forecast one to three years ahead, which are the horizons relevant for planning the purchases and deliveries of banknotes[17]. The forecasts are made with observations until 2015q4, i.e. seven, six, and five forecasts for the horizons of one, two, and three years ahead, respectively. The forecast errors of the models are reported by the mean absolute percentage error (MAPE), while comparisons are made with the root mean

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[17] Although estimations are made with quarterly observations, the evaluation of the projections is for the horizons one, two and three years ahead, which are the relevant ones for the planning of purchases and deliveries. Quarterly growth rates, instead of annual, are employed in order to explicitly model the stationarity. It can be evaluated separately when applying judgement to decide on delivery dates.

square error (RMSE) measures.[18] It is important to note that the forecasts made with the fundamental–based models are pseudo-out-of-sample such that actual values of the fundamental variables are applied. Hence, for practical purposes, where forecasts one, two, and three years ahead of the macroeconomic variables have to be used as input, the forecast will be affected by a higher degree of uncertainty.[19]

4.1. Aggregate demand for cash

To estimate the aggregate circulating stock of coins and notes we first estimate time series models defined in section 3.2. As shown in table A.2 in Appendix, the MAPE of the average forecast of the six time series model is around 2.1%, 3.8% and 5.5% for the 1, 2 and 3 years horizons of projection.

As shown in Figure 9, among the time series models that make the better forecasts are the one that simply utilizes the average of the same quarter the past five years (Mod.2) and the two seasonal ARIMA models (Mod.5 and Mod.6). The average forecast of the six models is usually better than those of the individual models, but only statistically significantly so in three cases (one-year-ahead versus Mod.3 and three-years-ahead versus Mod.3 and Mod.4). In none of the four situations in which individual models outperform the average prediction are the differences statistically significant.

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[18] $MAPE = \frac{1}{N}\sum_i^N \frac{|x_i - \hat{x}_i|}{x_i}$ and $RMSE = \sqrt{\frac{1}{N}\sum_i^N (x_i - \hat{x}_i)^2}$, where x_i and $\hat{x}_i$ are the actual data and the forecast, respectively. Statistical significance is evaluated with the Diebold and Mariano (1995) test applying the Harvey et al. (1998) small sample correction. Even though the tests are small sample corrected, a caveat is in place as very few observations are utilized for making the tests. Hence, references to statistical significance should be considered as merely illustrative.

[19] As a complementary exercise, the fundamental variables were predicted with AR(4) models and, interestingly, the projections were often better than those made with actual data, though not statistically significantly so. In this context it should be remembered that utilizing predictions of variables to make forecasts increases the uncertainty importantly.

Figure 9: RMSE of time series models[20]

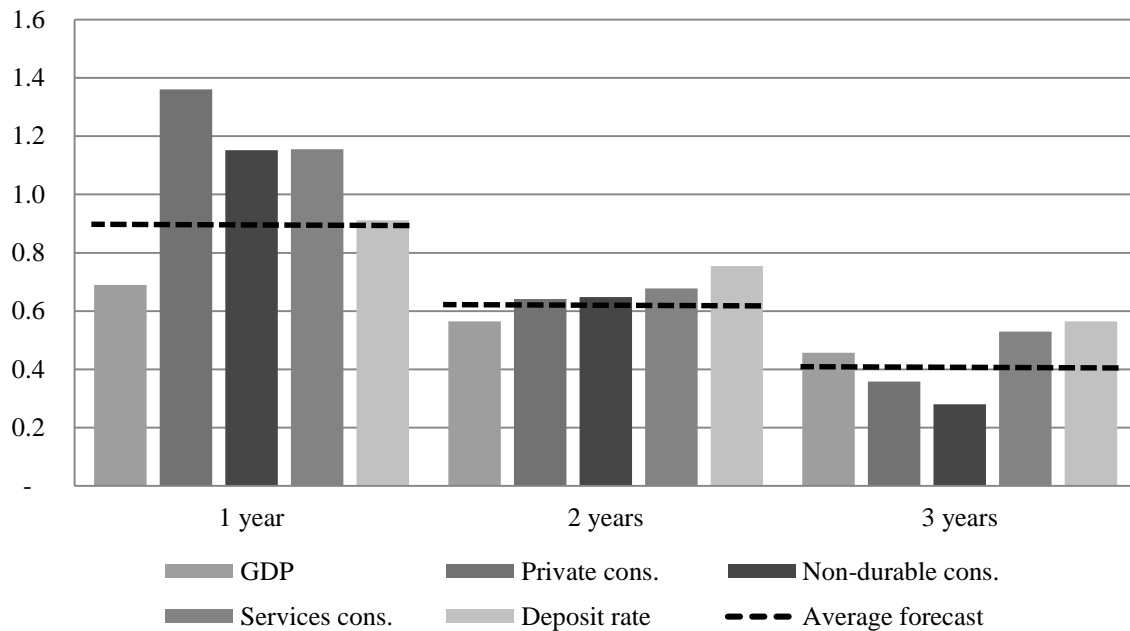(relative to RMSE of Mod.1)
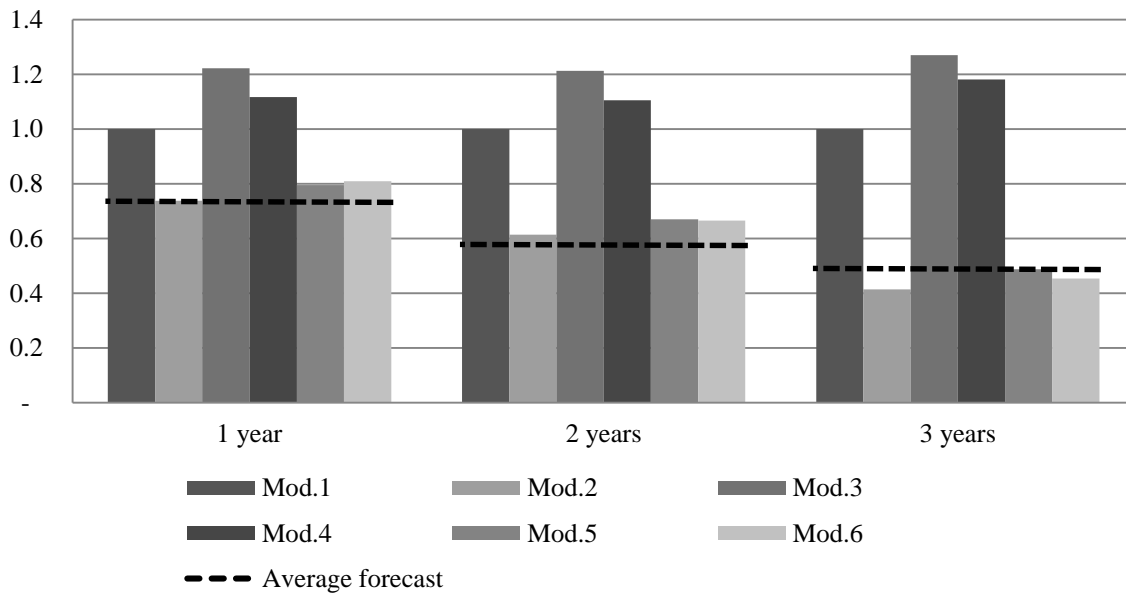
Source: Authors' elaboration.
Note: Average forecast is the average of the six models' forecast.

As mentioned earlier, the forecasts of the fundamental–based models defined in section 4.3— are made using the actual values of the macroeconomic variables and, hence, we are not considering the prediction error of the future values of fundamentals. The MAPE of the average fundamental-based model is 2.6%, 3.5% and 2.7% for estimates one, two and three years ahead, respectively (see table A.2 in the Appendix) and, hence, it yields better results than the time series models at the longer horizons. Particularly, for the three-year horizon the predictions are statistically better than the average time series predictions.

While the average forecast performs quite well, it is only statistically significantly better than that of the non-durable consumption model one-year-ahead and of the models that include GDP, private consumption, and services consumption three-years-ahead. In a couple of cases individual models perform better than the average, but the difference is not statistically significant in any of them.

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[20] The RMSE of Mod.1 are 213,623; 466,381; and 777,580 for the one, two, and three year horizon, respectively. This corresponds to 3.5%, 7.2%, and 11.3% of the outstanding stock.

Figure 10: RMSE of fundamental-based models
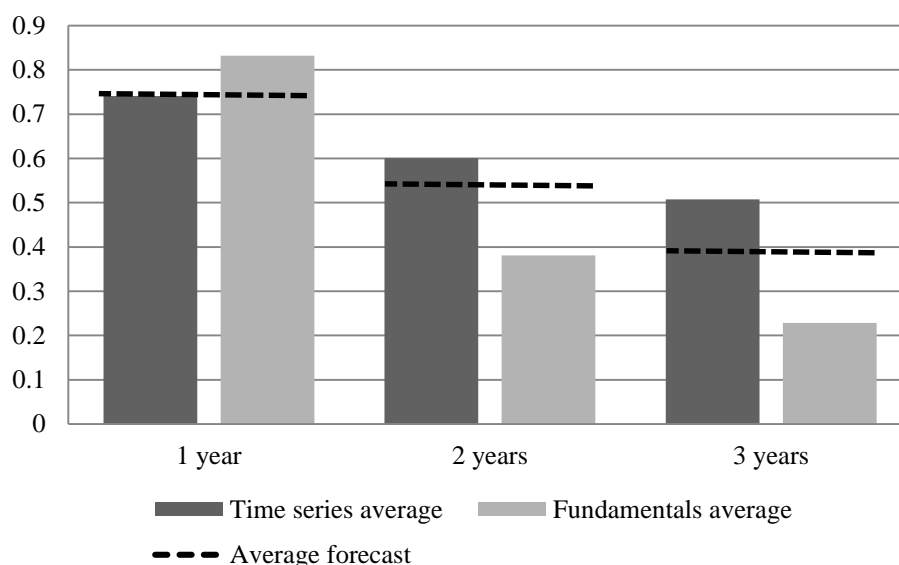
(relative to the RMSE of Mod.1)

Source: Authors' elaboration.
Notes: Average forecast is the average of the five fundamental based models' forecast.

Comparing the forecasts made with time series models against the ones made with fundamental-based models, the time series average performs better one-year-ahead, while the fundamental-based average is better for the longer horizons (Figure 11). Only with respect to the three-year-ahead projections, however, is the difference statistical significant. The average between the time series and the fundamental-based averages is statistically equivalent to each of them for the one and two year horizons, but it makes better three-years-ahead forecasts than the time series models and worse than the fundamental-based model. In the latter case the difference is statistically significant.

19

Figure 11: RMSE of average models

(relative to RMSE of Mod.1)

4.2. Demand by denomination[21]

Using the same time series model previously specified, we estimate and forecast the number of pieces of each denomination of coins and banknotes. Given that there is a great degree of heterogeneity in the performance of the models for each denomination —in the majority of the cases the average of the models is not the best one— we choose the model with the lowest sum of the RMSEs for one, two and three years of forecasting (see Table A.3 in the Appendix). As shown in Table A.4.a in the Appendix the mean absolute percentage error is relatively low across coins, but it is substantially higher for the $2,000 and $5,000 notes.

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[21] On October 26, 2016, the CBCh announced that it would stop minting the $1 and $5 coins, with the aim of increasing productivity. This will be effective as from October 26 of 2017, so projecting the future demand for these obsolete denominations would be pointless. Thus, they are omitted from the analysis. In the total circulating stock, however, they are included because their demand in terms of value is expected to be partially absorbed by other denominations.

To disaggregate the forecasts of the total stock of coins and banknotes, we estimate and forecast the weights of every denomination using an AR(4) model. With these estimated weights, the projections of fundamental-based models by denomination are obtained. As shown in table A.4.b in Appendix, the disaggregated projections of coins and banknotes have an average relative absolute error that varies much across horizons and denominations.

Figure 12 compares the RMSE of the best performing time series models by denomination with the disaggregated forecasts of the average of the fundamental-based model. With a couple of exceptions, the time series models have better forecasting performance. Hence, the exercise presented in the present paper suggests that models by denomination should be applied to guide the purchases of coins and banknotes in Chile, while aggregate forecasts, which are helpful for applying judgement before deciding on purchases, can benefit from the use of fundamental variables of GDP and private consumption.[22]

Figure 12: RMSE of Average Time Series Models

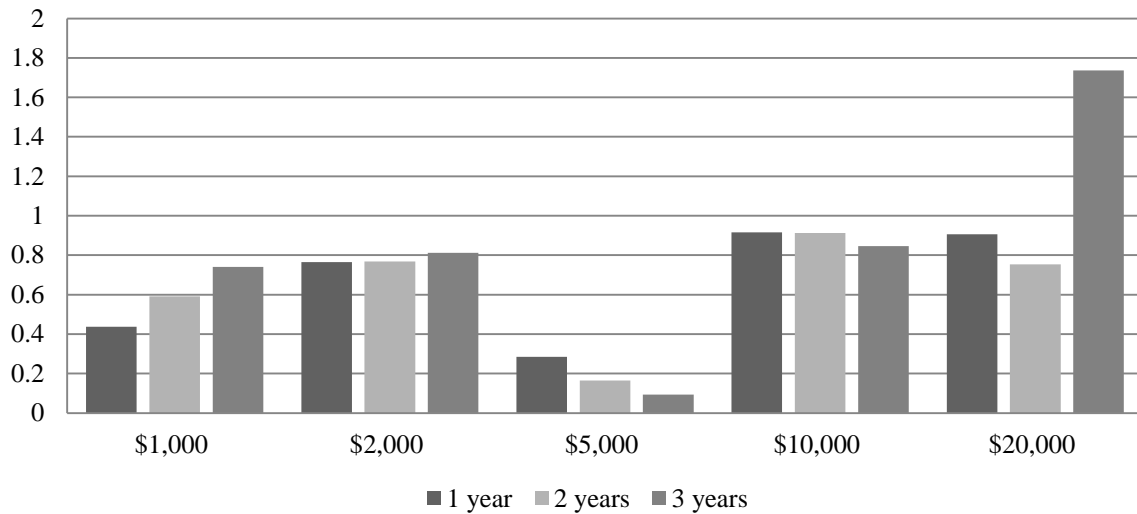(relative to the RMSE of Average Fundamental-Based Models = 1)

a. Coins

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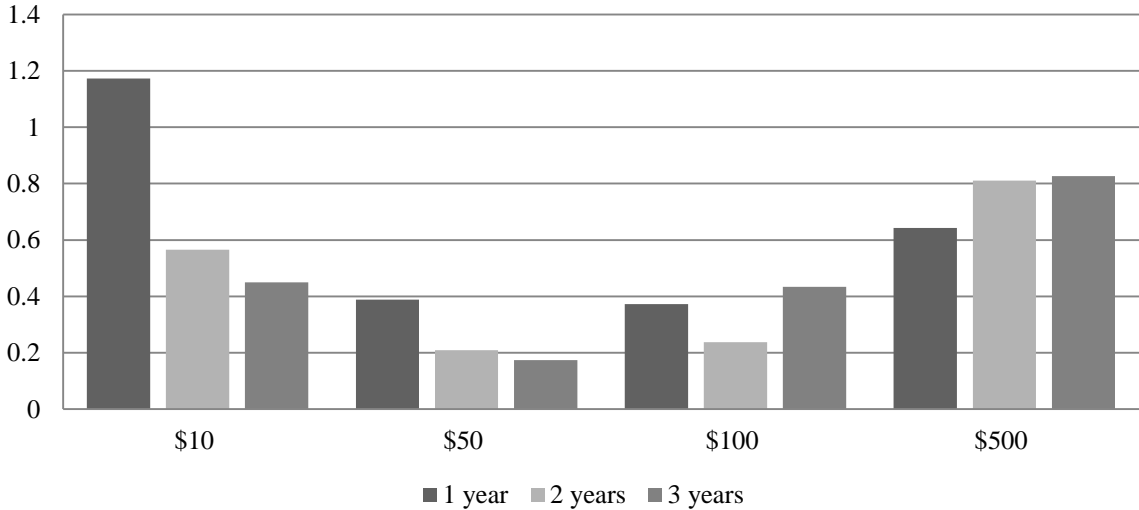
[22] Aggregating the forecasts of denominations does not yield better predictions than the aggregated models.

b. Notes

Bar chart showing values for denominations $1,000, $2,000, $5,000, $10,000, and $20,000 across three forecast horizons (1 year, 2 years, 3 years). Y-axis ranges from 0 to 2.

5. Conclusions and final remarks

In planning the purchases and future delivery of Chilean banknotes, it is important to have forecasts of the future demand as reliable as possible. In this context, the relevant horizons are one, two, and three years ahead as tenders are made for three years of delivery with the option to add additional pieces every year. While approximately 75% of the new banknotes are put in circulation to replace pieces that are no longer suitable for circulation, future projections of the circulating stock are indeed useful to guide the decisions of future purchases.

As a first exercise models of the aggregate circulating stock were estimated and evaluated with respect to forecast performance. While the main interest is to have models for denomination, aggregate forecasts are useful when applying judgement with respect to the forecast of denominations. While time series models seem to perform better for the one-year-ahead forecast, models with macroeconomic variables are better in the longer horizons, but the difference is only statistically significant for the three-year-ahead forecast. This suggests that forecasts of fundamental variables should be taken into consideration when deciding on orders for future delivery. With respect to the denominations, it is different time series models, or indeed the average, that make the superior forecasts. Hence, it is recommendable to take this fact into account when making the predictions.

The quest to find suitable models for forecasting never ends. Structural shifts and other changes of conditions may require updating existing models or even changing them for new ones. Usually it is advisable to have different models with different kinds of focus such that the input, not only of the actual forecasts but also of past errors, can be used in the judgement of making the final purchase decisions. In the present paper we presented time series models as well as some based on fundamental macroeconomic variables. Another direction would be to explore models in line with those utilized by Gerst and Wilson (2011) and Vale (2015), which take into account technology development and consumer taste. To what extent these characteristics can be employed formally in econometric models will depend on available data, and the development of such a framework is left to future research.

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Appendix

Figure A.1. Change in stock of banknotes by denomination

(millions of pieces)

$1,000

$2,000

$5,000

$10,000

$20,000
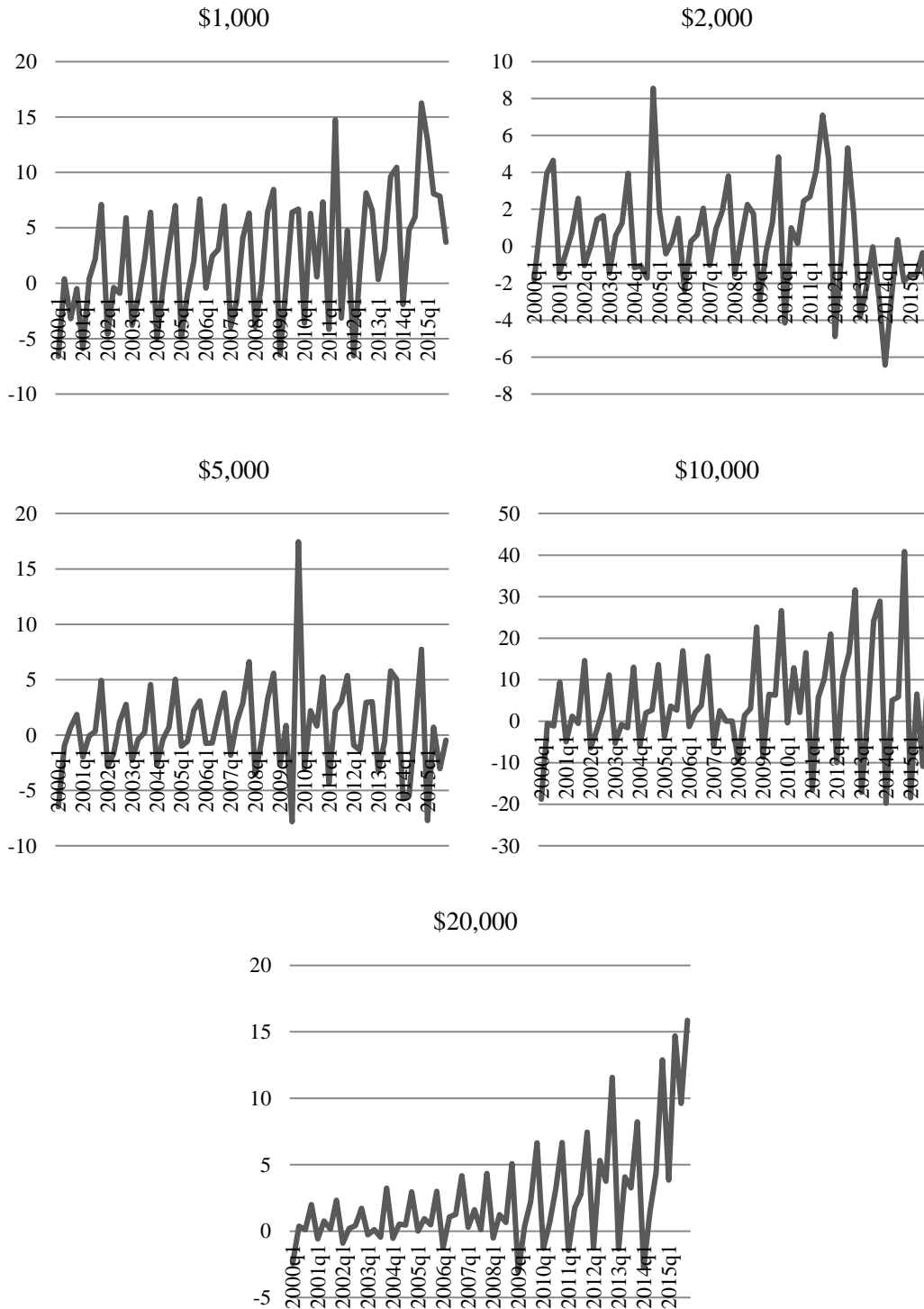
Source: Central Bank of Chile.

Figure A.2. Change in stock of coins by denomination

(millions of pieces)

$1

$5

$10

$50

$100

$500
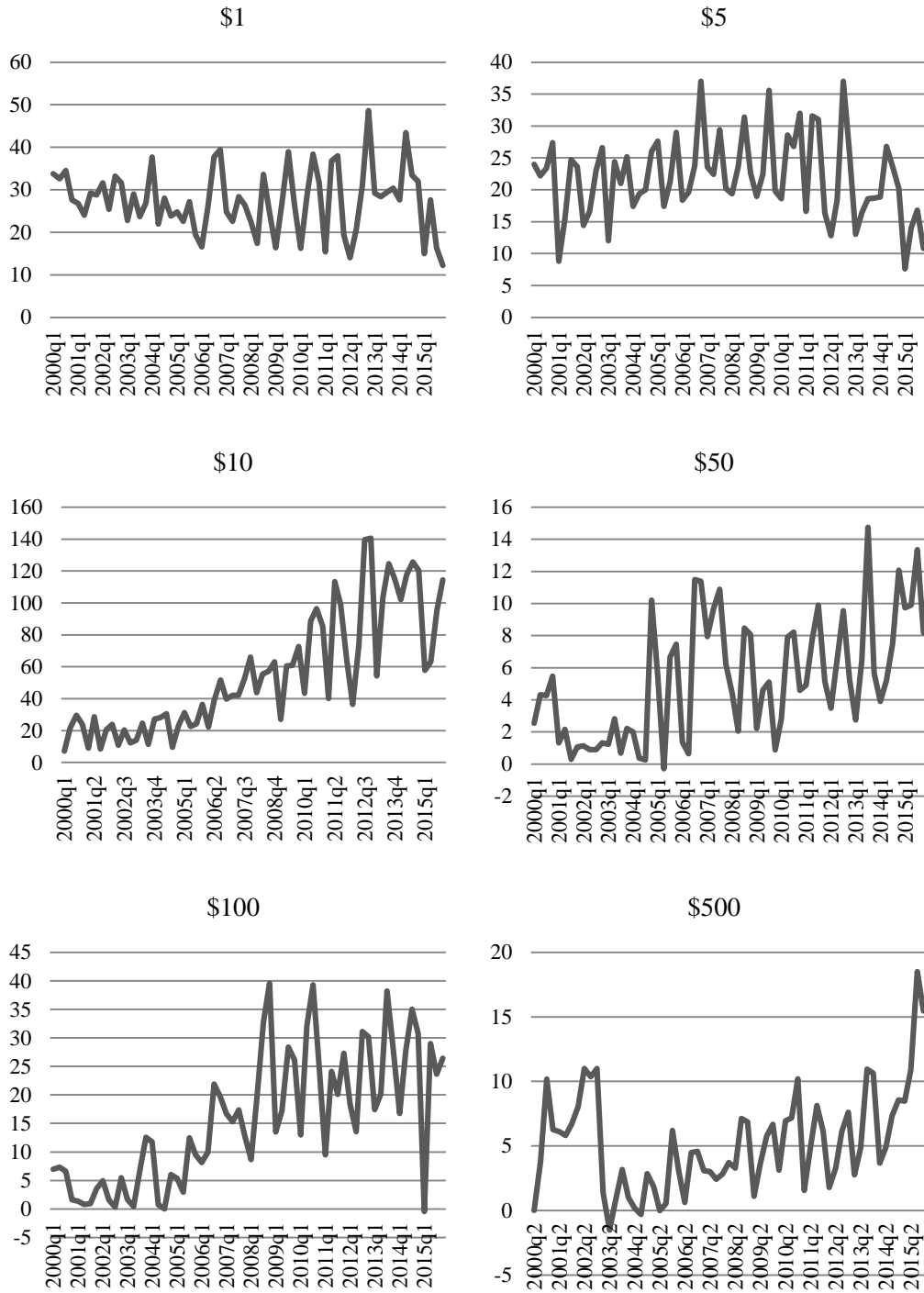
Source: Central Bank of Chile.

Figure A.3. Demand for and supply of banknotes by denomination

(millions of pieces)

$1,000

$2,000

$5,000

$10,000

$20,000
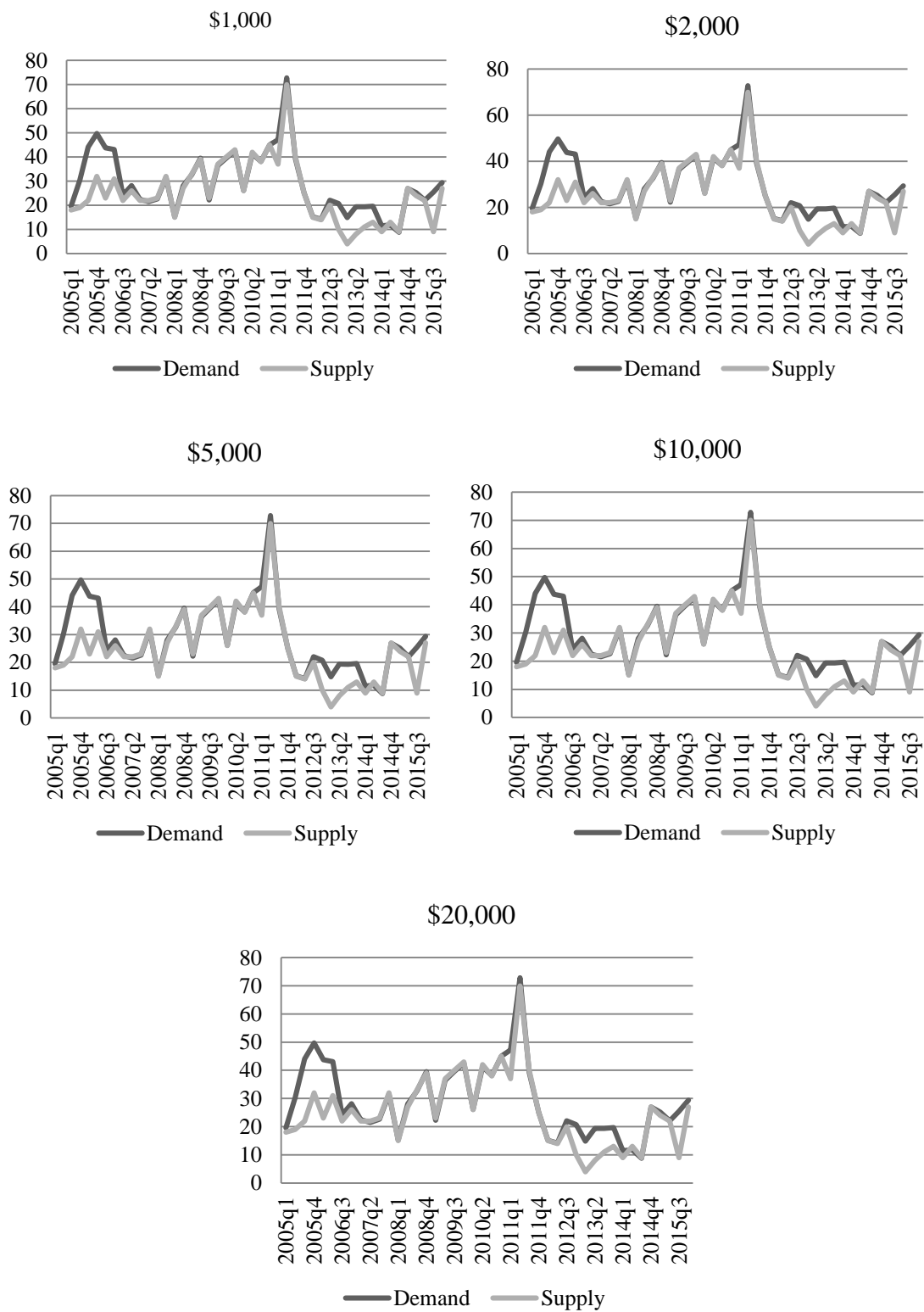
Source: Central Bank of Chile.

Figure A.4. Demand for and supply of coins by denomination[23]

(millions of pieces)

$1

$5

$50

$100

$10

$500
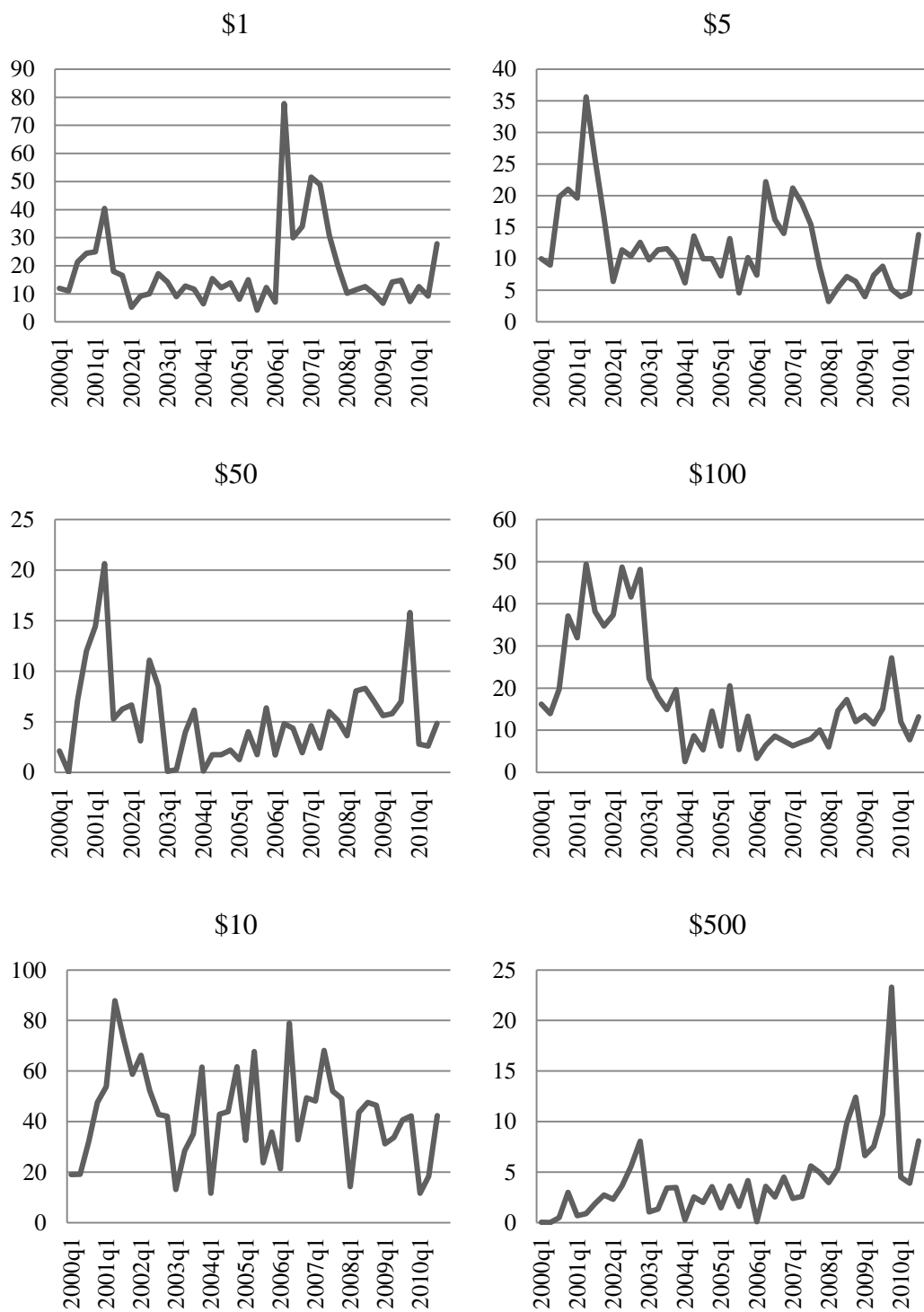
Source: Central Bank of Chile.

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[23] Since the quantity demanded is always delivered, demand and supply series are equal.

Table A.1. Bivariate VAR Models

a. Activity variables

Stock	GDP		Private cons.		Durable cons.		Nondurable cons.		Service cons.	
Equation	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value
Lag 1	0.19	0.32	-0.22	0.48	-0.05	0.43	-0.21	0.29	-0.14	0.66
Lag 2	-0.11	0.58	0.27	0.40	0.07	0.29	-0.11	0.58	0.48	0.12
Lag 3	0.49	0.01	-0.33	0.31	0.04	0.55	-0.41	0.04	-0.13	0.65
Lag 4	-0.50	0.01	-0.22	0.47	-0.23	0.00	0.20	0.31	-0.24	0.41
R-squared	0.8269		0.8054		0.8412		0.8153		0.8072	

b. Interest rate

Stock	Deposit rate	
Equation	Coef.	p-value
Lag 1	-0.21	0.10
Lag 2	-0.17	0.25
Lag 3	-0.40	0.00
Lag 4	-0.36	0.00
R-squared	0.837	

Source: Authors' elaboration using data from the Central Bank of Chile.

Table A.2. Mean Absolute Percentage Error for Total Stock of Banknotes and Coins

H	Time series average	Fundamentals average	average
1 year	2.096%	2.374%	2.017%
2 years	3.821%	3.299%	3.356%
3 years	5.493%	2.999%	4.246%

Source: Authors' elaboration using data from the Central Bank of Chile.

Table A.3. Time Series Model with Lowest RMSE by Denomination

H	$10	$50	$100	$500	$1k	$2k	$5k	$10k	$20k
1 year	M6	M6	Av	M2	M1	M4	M3	M1	M2
2 years	M1	Av	Av	M2	M1	M4	M4	M2	M4
3 years	M2	Av	Av	M2	M6	M4	M4	M2	M4
Total	M2	Av	Av	M2	M6	M4	M4	M2	M4

Source: Authors' elaboration.
Notes: M: model. Av: Average of the six models. Total: Model with the lowest sum of RMSE in the 3 years.

Table A.4. Mean Absolute Percentage Error by Denomination

a. Selected time series model

H	$10	$50	$100	$500	$1k	$2k	$5k	$10k	$20k
1	1.38%	1.42%	1.59%	2.99%	2.70%	11.72%	7.09%	4.22%	5.15%
2	2.30%	2.04%	2.13%	4.44%	4.65%	27.85%	8.89%	6.15%	6.36%
3	3.40%	2.76%	5.75%	5.30%	6.96%	42.97%	11.69%	7.22%	7.76%

b. Average fundamental-based model

H	$10	$50	$100	$500	$1k	$2k	$5k	$10k	$20k
1	**1.37%**	4.20%	4.43%	4.65%	5.37%	18.68%	27.54%	5.03%	6.00%
2	4.76%	11.11%	8.53%	5.08%	7.23%	39.42%	76.78%	6.48%	8.04%
3	7.90%	17.95%	13.97%	7.47%	9.63%	48.59%	134.24%	8.90%	7.77%

Source: Authors' elaboration.
Note: Bold indicate that the number is lower than in table a.

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